

Advertising Competition with Associative Memory*

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April 2026

*This work was funded by Fundação para a Ciência e a Tecnologia (UIDB/00124/2020, UIDP/00124/2020 and Social Sciences DataLab - PINFRA/22209/2016), POR Lisboa and POR Norte (Social Sciences DataLab, PINFRA/22209/2016). Email addresses: pedro.gardete@novasbe.pt and villas@haas.berkeley.edu.

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Abstract

This paper studies advertising competition when consumers rely on associative memory. Memory is modeled as a network in which brands become linked to value-relevant concepts through repeated exposure, and advertising operates by triggering recall. In this environment, communication shapes consideration sets by determining which brands and attributes are jointly brought to mind. In contrast to belief-based persuasion models, communication in our setting is effective even when it conveys little or no new information, as long as it succeeds in inducing recall of the brand together with a value-relevant concept.

The paper shows that firms face a trade-off when emphasizing product benefits. Associating a brand with value is necessary to encourage consideration, but doing so may also activate competing brands through shared memory links, intensifying competition. As a result, firms may optimally rely on uninformative advertising: once associations are established, simple brand reminders can sustain relevance while limiting competitive spillovers. When advertising effectiveness is intermediate, however, firms are forced to emphasize value, leading to joint recall and heightened competition. As a result, firms' profits depend non-monotonically on advertising effectiveness.

We further show that utility-irrelevant associations – such as colors, mascots, or symbols – may allow firms to circumvent this trade-off: By linking these cues to value during earlier communication, firms can later use them to trigger recall without activating competing brands, thereby softening competition and increasing profits.

*And as I ripped it open, my father tousled my hair, and forever,
his love and the chocolate were tied together.
That's the story we're going to tell.
Hershey's is the currency of affection.*
— Don Draper, Mad Men (S6.E13)

1 Introduction

Advertising frequently promotes associations between brands and ideas. In the passage above, the fictional character Don Draper pitches the Hershey's brand as the currency of affection. Brand associations are common in advertising markets: Red Bull “gives you wings”, Volvo is “for life”, and Disney parks are “the happiest place on earth.”

These pairings are consistent with insights from cognitive psychology. The associative memory literature predicts that linking a brand to subjective or objective benefits strengthens both its storage and its likelihood of recall in consumers' minds (see, for example, Anderson, 1983; Rumelhart, McClelland, Group, et al., 1986). At its core, associative memory represents concepts not in isolation, but as nodes in a network, where each node's activation depends on the strength of its connections to others. The mere mention of a concept may trigger the activation of related ones and increase their relevance for decision making. As a result, what comes to mind at the moment of choice depends on the network of associations stored in memory.

Marketers have long recognized this principle. As expressed in the well-known adage by Trout and Ries (1986), “Positioning is not what you do to a product. Positioning is what you do to the mind of the prospect. That is, you position the product in the mind of the prospect.” The examples above illustrate this idea: Red Bull is linked to energy, Volvo to reliability, and Disney parks to happiness – all of which are desirable ‘positions’ in the minds of consumers.

This paper investigates how marketing communications can be used to influence consumers who exhibit associative memory. Because this persuasion channel can affect which aspects are accessible at the time of decision making, it can have a real impact on market outcomes. In our setting, strategic marketing communication is employed by firms to influence how their brands are encoded in memory and subsequently recalled. Upon exposure to advertising, associative memory governs which brands and benefits come to mind and whether they warrant consideration. Overall, firm communication can be thought of as shaping consumers' consideration sets, subject to the rules of associative memory. The formalism we employ draws on Hebbian learning (Hebb, 2005), in which “cells that fire

together, wire together”. For instance, the co-occurrence of a brand cue and a concept associated with customer value (e.g., quality, happiness, status) leads to a systematic increase in the weight linking those two concepts in memory. This framework offers a tractable way to endogenize the evolution of consideration sets. Hebbian dynamics translate strategic communication choices into changes in the consumer’s internal memory network, and thus into the probability that each brand enters the consideration set at the time of purchase.¹

Arguably, memory-based persuasion offers several advantages over existing frameworks in the marketing and economics literatures. First, it explains the empirical observation that much of persuasion effectiveness arises from invoking arguments already known to the receiver (see DellaVigna and Gentzkow, 2010). In such cases, communication influences behavior not by providing new information, but by directing attention to specific elements.

Second, the persuasion channel we model does not rely on the receiver’s ability to engage in deep deliberation about the incentives of the sender. This is a common criticism of signaling models, including costly signaling (Spence, 1978; Milgrom and Roberts, 1986), cheap-talk communication (Crawford and Sobel, 1982), and Bayesian persuasion (Kamenica and Gentzkow, 2011). In belief-based models of persuasion, receivers are required to map messages to sender types, a task that requires an understanding of the incentives of each type, as well as their distribution. It is likely that in some real-world settings, individuals are unable to meet this requirement. Moreover, in many settings receivers are influenced despite facing little uncertainty about the sender’s type or incentives, calling into question belief-based mechanisms.² In contrast, in the model presented here advertising does not change what consumers believe – it changes what they remember at the moment of choice.

We consider the case in which two firms compete for the business of a customer. To induce a purchase, they decide the aspects they would like to associate with their respective brands. We first consider the case in which firms can associate their brands with a relevant benefit, which we refer to as value. The concept of value admits several interpretations. For example, it may denote a benefit, idea, or attribute that the consumer deems important for meeting her wants and needs, such as quality, competence, convenience, customer service, etc. When communication emphasizes a brand in association with value, consumers become

¹Consideration sets are a well-developed concept in the marketing literature (see, for example, Hauser and Wernerfelt, 1990; Mehta, Rajiv, and Srinivasan, 2003; Iyer, Soberman, and Villas-Boas, 2005; Van Nierop, Bronnenberg, Paap, Wedel, and Franses, 2010; Hauser, 2014; Honka, Hortaçsu, and Wildenbeest, 2019). The approach presented here provides a microfoundation of how firms can influence consumer consideration sets through communication.

²In the cheap-talk literature, Chakraborty and Harbaugh (2010), Gardete and Bart (2018), Lipnowski and Ravid (2020) consider cases in which the receiver is only required to understand that the sender would like to induce a ‘conversion’, such as a sale. However, their frameworks still require that the receiver understands the probability distribution of a state decided by nature, often relying on a mapping from a set of states to each possible message.

more likely to include it in the consideration set.

In this context, firms are assumed to offer value sought by the consumer, but this information must be successfully encoded in the consumer’s memory to enable inclusion in the consideration set.³ To clarify the possible operationalizations of value, consider for example a customer who has only a vague knowledge about a computer brand, without having associated it with attributes such as performance or reliability. She may discard the brand and opt for a well-known brand instead, whose value has already been established. Relatedly, consider the case of a consumer with needs or wants related with exercise (e.g., dumbbells) or better rest (e.g., mattress). The consumer may discard a recollected brand if she does not recall that the firm carries those products, or that it can provide them competently. For example, Worten, a Portuguese retailer, runs the following advertising slogan: “[Worten] Has everything and who knows what else.” Because Worten is primarily known for selling household appliances and technology products – and initially communicated that focus – the fact that it also offers dumbbells and mattresses may not come to the consumer’s mind at the time of search or purchase.

Finally, value may also be interpreted as a want or need. For example, an advertisement for an air freshener may be reinforced by a product demonstration highlighting the actors’ positive reactions. This may prompt the consumer to recognize a relevant need that warrants attention to the marketplace. Absent such recognition, the consumer may assign little value to the available offerings and not buy anything.

In the model considered, firms compete by engaging in two communication stages, which we denote as positioning and advertising. In the positioning stage, firms prime the consumer’s memory by establishing initial associations between their brands and value, and in the advertising stage, firms’ messages induce recall in order to promote a conversion. We find that firms always benefit from associating their brands with value in the positioning stage. Despite the crucial role of value in trade, however, our results show that firms may benefit from not emphasizing value in the advertising stage, and instead opting for uninformative advertising, in which their brands are not associated with any benefits. Specifically, when advertising is very effective (e.g., when it is inexpensive to induce recall), firms are better off engaging in uninformative communication by mentioning their brands without referencing value. In doing so, they prevent the activation of a memory backchannel that links competing

³In other words, communication is required to establish awareness of the fact that, for example, a brand operates in a given industry or provides a given benefit (e.g., Schmalensee, 1986). This assumption is also implementable in the paradigm of classical information transmission models. For example, assume the consumer’s prior assigns negative utility to new products. Through (credible) communication, firms can reveal relevant signals that induce an upward revision to prior utilities, and consideration sets are composed of those products whose expected utilities are positive.

brands in the consumer's mind, through the value concept. In such cases, the mention of one brand in association with value could bring to mind its competitor, increasing competition and lowering profits. When advertising effectiveness is intermediate, firms become worse off if they do not associate their brand with value: communicating the value association becomes necessary to ensure brand consideration, but at the cost of bringing the competitor's brand to mind as well. As a result, competition intensifies and profits decline. Finally, when advertising effectiveness is low, the strength of the memory backchannel diminishes, and competition softens once again. Hence, the model predicts a U-shaped relationship between advertising effectiveness and firm performance.

As for consumer welfare, communication always generates value insofar as it is necessary for trade to occur. However, it is most beneficial when advertising effectiveness is intermediate. In this range, firms must associate their brands with value during the advertising stage to ensure consideration. Doing so brings both brands to the consumer's mind, intensifying competition and improving the consumer's ability to evaluate alternatives, ultimately enhancing welfare.

We also consider the case in which firms may also associate their brands with irrelevant concepts or attributes, such as symbols or colors. This case is different from the examples provided earlier, in which firms associate their brands with specific benefits. Unlike attributes that actually provide utility to consumers, utility-irrelevant concepts are plentiful and easily accessible. For example, firms may associate their brands with animals (e.g., Aflac ads feature a white goose, GEICO ads feature a gecko), colors (e.g., UPS is strongly associated with the color brown, Coca-Cola leverages its association with the color red), symbols/shapes (e.g., Target often features a red bullseye, Nike makes strong use of its 'swoosh' logo), etc. Analysis of the augmented game reveals that associating a brand with utility-irrelevant concepts can meaningfully affect market outcomes. Specifically, firms are able to alleviate the intense competition that arises when advertising effectiveness is intermediate by advertising the utility-irrelevant concept. To do so effectively, firms must first associate both their brand and the irrelevant concept with value during the positioning stage. Then, in the advertising stage, communicating only the irrelevant attribute becomes a successful strategy for entering the consumer's consideration set. Because the utility-irrelevant concept is only indirectly linked to value, this strategy avoids activating the memory backchannel across competing brands. As a result, a firm's advertisement no longer brings the rival's brand to mind, thereby softening competition and increasing profits. In other cases, firms become better off relying on all available cues. This outcome generates cross-recall in equilibrium, such that a firm's advertisement will induce consideration of its rival's brand with positive probability. Overall, the augmented game shows that firms can use seemingly irrelevant

brand associations to remain salient to consumers without intensifying competition. As for consumers, they are unambiguously worse off by the use of utility-irrelevant concepts.

There are relatively few formal models linking memory and persuasiveness. Bhatt (2012) develops a neurally inspired model in which advertising shapes product evaluation through associative network dynamics. In that framework, products and experiences are encoded as patterns in a network where connections between characteristics are strengthened through co-occurrence. When a consumer evaluates a product, its observable characteristics activate the network, which then converges to a stable pattern of perceived characteristics. Advertising operates by altering these associations, thereby creating complementarities across attributes and allowing observable features to activate abstract or social characteristics (e.g., fashion or prestige). As a result, advertising effectively transforms the mapping from objective to perceived attributes, generating equilibrium phenomena such as spillovers and brand dilution in competitive environments.

The framework presented here differs along a few key dimensions. First, the paper focuses explicitly on firms' strategic choice of communication content – namely, which concepts to associate with their brands – rather than on the formation of associations through exposure. Second, we allow these associations to involve attributes that may be irrelevant for intrinsic consumption utility, thereby capturing uninformative advertising in a tractable way. Third, and most importantly, we do not model advertising as transforming the perceived attribute vector itself. Instead, advertising operates by affecting which pieces of information become salient at the moment of evaluation – that is, what is retrieved from memory and what is not. In this sense, the approach presented here remains closer to the standard economic tradition: preferences are defined over attributes, and advertising operates through selective accessibility and retrieval rather than through endogenous perception formation.

Sahni and Yang (2024) analyze a context in which firms compete in a probabilistic recall process: advertising strengthens the association between a brand and the purchase context, thereby increasing the likelihood that the brand is retrieved from memory. A key competitive force is memory interference, whereby a firm's advertising reduces the recall probability of competing brands. While that framework shares the emphasis on memory-based competition, it abstracts from the content of communication. In contrast, the analysis presented here places the content of advertising at the center of the strategic problem, showing how different types of associations – particularly those unrelated to intrinsic utility – affect equilibrium outcomes.

Shapiro (2006) provides a microfoundation for persuasive advertising based on limited memory and recall. In that model, consumers accumulate past consumption experiences and, at the time of purchase, base their decisions on a recalled subset of these experiences. Ad-

vertising affects behavior by altering the recall process – either by increasing the likelihood that favorable experiences are remembered or by biasing which experiences are retrieved. This mechanism operates through belief formation about product quality. In contrast, this paper abstracts from belief distortion and instead focus on associative memory and consideration: advertising links brands to concepts, including those unrelated to intrinsic utility, and thereby shapes which options and attributes are brought to mind. As a result, the framework highlights a distinct channel of persuasion – operating through consideration and communication rather than through beliefs about product performance.

Finally, Mayzlin and Shin (2011) propose a mechanism that predicts the use of uninformative communication. In that model, firms sell multidimensional products to bandwidth-limited consumers. Because limited bandwidth prevents a high-quality firm from separating from a medium-quality firm through attribute claims alone, it may instead choose uninformative advertising. Such advertising policy induces consumer search, which probabilistically reveals quality. Since high-quality firms benefit more from scrutiny than lower-quality firms, only the high type is willing to trigger search, allowing separation without relying on disclosing advertising expenditures. This paper proposes an alternative explanation for the occurrence of uninformative advertising: when consumers exhibit associative memory, providing information through value-based cues may activate competing brands in memory, thereby intensifying competition. Uninformative advertising, on the other hand, can be used to advertise one’s own brand without necessarily bringing the competitor’s brand to mind.

The next section briefly introduces and operationalizes associative memory, and explains how it can be used to conceptualize brands and brand associations. Section 3 considers the case of competition in which firms communicate by contemplating in informative and uninformative content. Section 4 considers the case in which firms may also include utility-irrelevant concepts in their communication. Section 5 provides concluding remarks.

2 A Model of Associative Memory

There exists a long literature on associative memory, a significant part relying on the concepts of Hebbian learning and Hopfield networks. The typical focus of this literature is to understand the properties of memory models, including their convergence rates and memory capacity (see, for example, Shah and Miyake, 1999; Hertz, 2018; Malmberg, 2024, for formalizations of different memory models).

Because the goal here is to investigate the effects of associative memory in market settings, we opt for parsimonious specifications that can nonetheless capture key features. Following

basic Hebbian learning, the state of a consumer’s memory at time t can be defined by a matrix W_t , each component (row and column) originally related to a concept that can be co-activated with others. Consider the following 3×3 matrix capturing an individual’s recall weights for the animals ‘orcas’, ‘sharks’, and ‘dogs’:

$$W_t = \left[\begin{array}{ccc|c} 5 & 0 & 0 & \textit{orcas} \\ 0 & 5 & 0 & \textit{sharks} \\ 0 & 0 & 10 & \textit{dogs} \end{array} \right] \quad (1)$$

The matrix above is composed of positive numbers. Each element in the main diagonal captures the recall strength of each given concept stored in memory, and the off-diagonals capture the strength of co-activations (currently none exist). Matrix W_t can hold positive values, with zero representing an absence of memorization of a concept when located in the main diagonal, or the absence of an association between two concepts in the off-diagonals. Above, ‘dogs’ is the best memorized concept, and no associations have been formed in the individual’s mind (i.e., all elements off the main diagonal are zero).

Memory models feature two central operations: recall/retrieval and storage. Once presented with stimuli, information that may be relevant for decision making is retrieved. Moreover, stimuli are also stored in memory, leading matrix W_t to evolve over time. We now specify the functional forms of each of these operations.

Recall/Retrieval. When an individual is exposed to stimulus m_t , a specific memory r_t is induced. While the individual’s memory state is captured by W_t , her actual recall is given by vector r_t . Thus, even if an individual has been exposed to several stimuli over time, at time t only a subset of those experiences may come to mind. The literature on associative memory considers several specifications for the recall operation, often emphasizing iterative and feedforward recall. We specify a feedforward recall procedure, such that upon receiving stimulus or message m_t in state W_t , recall r_t is produced. Continuing with the animal associations example, suppose the individual with memory state W_t subsequently watches a marine wildlife documentary featuring both orcas and sharks. The stimulus can be encoded by the corresponding animals’ presences via the binary vector $m_t = \left[1 \ 1 \ 0 \right]'$. This

stimulus induces the recall vector r_t , according to the feedforward rule:⁴

$$r_t = W_t \cdot m_t \tag{2}$$

$$= \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 10 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \tag{3}$$

$$= \begin{bmatrix} 5 \\ 5 \\ 0 \end{bmatrix} \tag{4}$$

Vector r_t contains recall strengths associated with each of the concepts. We adopt the classical assumption that a concept is recalled if it equals or surpasses a threshold value θ . As the value of θ increases, memories need to be reinforced more often (or with more intensity) until they can be retrieved. In the example above, when $\theta \leq 5$, the mention of orcas and sharks will lead the individual to recall what s/he knows about these two animals. Moreover, for any (positive) value of threshold θ , the marine wildlife documentary will not induce recall of dogs, despite the fact that this animal exhibits the highest recall strength among the three. In market contexts, a higher value of θ can be interpreted as advertising being less effective in influencing consumers, or roughly equivalent, that it is more costly.

To summarize, memories are produced in agents by stimuli. They are triggered automatically and are a function of the agent’s internal memory state (which is not accessible to her) and the content of the stimulus. The induced recollection then becomes available as working memory for the agent’s decision making.

Storage. In addition to inducing recall, stimuli also subsequently affect the internal memory via a storage step. We adopt the Hebbian memorization rule:

$$W_{t+1} = W_t + m_t \cdot m_t' \tag{5}$$

where W_t is the contemporaneous memory state, and W_{t+1} is the state obtained after the individual is presented with stimulus m_t . The rule above is intuitive in that W_{t+1} is obtained by adding the outer product of the stimulus vector.⁵ In the working example above, in

⁴Iterative recall is more complex and is primarily used to investigate the convergence properties of memory systems, which is outside the scope of this paper.

⁵On a related note, the Hebbian storage step we assume is relatively similar to the Bayesian updating rule of the covariance matrix of the normal distribution with a conjugate prior. Given the two-period setup, forgetting is not considered (e.g., Villas-Boas and Villas-Boas, 2008). Consumer forgetting can be incorporated easily into the analysis, and the results remain robust as long as it is not too strong.

addition to inducing recall, watching the marine wildlife documentary will also affect the individual’s internal memory state, according to:

$$W_{t+1} = W_t + m_t \cdot m_t' \tag{6}$$

$$= \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 10 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 0 \end{bmatrix} \tag{7}$$

$$= \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 10 \end{bmatrix} + \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 6 & 1 & 0 \\ 1 & 6 & 0 \\ 0 & 0 & 10 \end{bmatrix} \tag{8}$$

The documentary reinforces the recall strengths of orcas and sharks, increasing them from 5 to 6. Moreover, the fact that the animals are featured together creates a novel association in the individual’s mind, as represented by the off-diagonal elements. In fact, one can verify that a subsequent stimulus featuring only orcas ($m_{t+1} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}'$) will induce recall of sharks when $\theta \leq 1$, but never of dogs.

Unlike traditional memory models, this framework allows the main diagonal of the memory state matrix to feature non-zero elements. This allows for self-reinforcement, which means that a concept is made more accessible the more it is stimulated. This specification differs from the most common representation of Hopfield networks, in which the diagonal of W_t is set to zero, ruling out self-reinforcement and persistence. In contrast, this specification preserves the standard economic intuition that a mention of a concept may also reinforce its recall even in the absence of additional associative input. This idea is in line with common specifications in the marketing and economics literatures, in which stimuli can reinforce the utility or likelihood of a single action (e.g., Nerlove and Arrow, 1962).

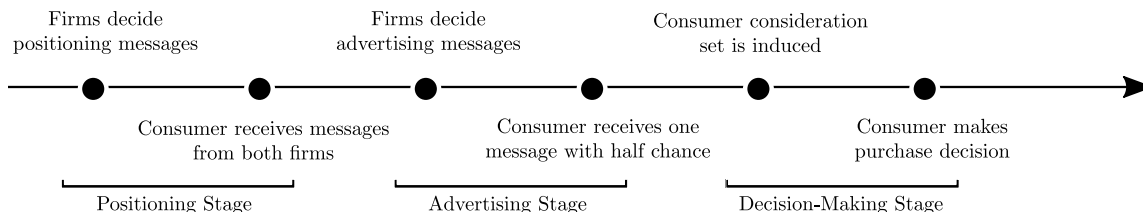
To summarize, we consider a memory model in which the individual’s memory state at time t is described by matrix W_t . When presented with a stimulus vector m_t , a memory r_t is produced and can be used for decision making. Following recall, storage takes place in the form of W_{t+1} , obtained by adding the current memory state to the outer product of the stimulus m_t .

3 Informative and Uninformative Advertising

Preliminaries. Consider the following stylized setting that captures the key competitive forces in a new market where advertising operates through associative memory: Two firms, indexed by 1 and 2, compete for a consumer with associative memory. The timing is

organized into three stages, as depicted in Figure 1.

Figure 1: Timing of the game



Because associative memory requires priming to function, let us consider the case in which firms communicate with the consumer in two consecutive stages. In the first stage, denoted as the positioning stage, each firm sends a message to the consumer. This stage can be interpreted as marketing positioning, reflecting the firms’ efforts to establish baseline associations in the consumer’s mind. In the second stage, denoted as the advertising stage, each firm again sends a message to the consumer, which may be different from the message sent previously.

Let us assume that both messages are received by the consumer in the positioning stage, so that her initial memory state holds a full set of associations between the brands and the remaining concepts. In the advertising stage, a single advertisement exposition is sufficient to induce recall and enable conversion. Therefore, at that stage, let us assume that one, and only one, of the advertising messages is received by the consumer with uniform probability. While the model is robust to different assumptions – including stochastic exposure in the positioning stage and complete coverage in advertising stage – this structure is sufficient to capture the main market tensions while keeping the analysis relatively simple.

The role of the advertising exposure in the second stage is to attempt to induce a conversion by the consumer. As discussed in the introduction section, a conversion requires the co-activation of at least one brand along with the concept of value. If value were not activated, a brand could well be recalled but still fail to make it to the consideration set, due to a perceived lack of relevance. Equivalently, the consumer could attribute worth to the brand, but fail to recognize her own need or want. Therefore, the communication game may result in the consumer’s consideration set holding no brands, one brand or two brands. In the decision-making stage, the consumer selects the option that yields the highest utility and outcomes are realized.

We assume that firms prefer to be considered in isolation rather than together with their rival. This assumption is sufficient to characterize firm payoffs and allows us to be agnostic about the specific type of competition taking place. The results require only that firms

prefer to be considered by the consumer exclusively with half chance, than together with their competitor with certainty. This is a common result in models of competition: Due to strategic interaction, the profits resulting from duopoly settings are typically less than half the corresponding monopoly profits (e.g., Hotelling, Cournot, Bertrand). Let the expected profit earned by a firm that is exclusively remembered by the consumer with half chance be given by $\frac{\pi^M}{2}$. If an advertisement induces a consideration set with both firms, each firm then earns an expected profit of π^C , such that $\pi^C < \frac{\pi^M}{2}$.⁶

In the context of communication games, the question sometimes arises of how to measure consumer welfare. For example, could ‘fooled’ consumers fare better than others, based on their incorrect beliefs? Because in this setting communication affects choice via consideration set formation, there is no additional difficulty in measuring consumer welfare. We assume that larger consideration sets offer more choice while inducing more competition, so that $CS^C > CS^M$. In addition, define the surplus of considering no alternatives as $CS^0 \equiv 0$, such that $CS^C > CS^M > CS^0$. Finally, this welfare ordering is consistent with an informative view of advertising, in which communication is necessary to inform the consumer of available options and potential gains from trade.⁷

Model. Let W be a 3×3 matrix summarizing the consumer’s memory state, initialized by the firms’ positioning choices. Because associative memory is most relevant for new brands, let us normalize the recall weight of the brands to 1 and denote the recall weight of value as $\phi > 1$. The setting considered is one in which the goal of firms is to strengthen their brands by associating them with the benefits sought by consumers.⁸

Let firms’ decisions to associate their own brands with the value concept be denoted as $x_t^j \in \{0, 1\}$, where $j \in \{1, 2\}$ denotes the firm and $t \in \{0, 1\}$ denotes the stage of the communication game (positioning and advertising), so that $x_0^1 = 1$ means that firm 1 decided to associate its brand with value in its positioning message, and $x_0^1 = 0$ means that firm 1 prefers to position its brand without associations. For illustration purposes, consider the case

⁶For illustration purposes of a full market specification, consider the utility from consuming brand j as $u_j = v \cdot \mathbf{1}(\text{Brand } j \in \text{Consideration Set}) - t \cdot d_j$, where v is the gross consumption utility, t is a transportation cost and d_j is the disutility of purchasing from brand j , which may be a random variable. This specification yields a market with differentiated demand a la Hotelling, with consideration sets induced by the communication game.

⁷As discussed above, the setting considered is one in which firms generate value for consumers, but this value is not directly observable and must be communicated to the market. (Alternatively, consumers may have latent wants or needs of which they must be reminded in order to consider a purchase.) This assumption is consistent with the intuition in Milgrom (1981), where the burden falls on firms to credibly disclose information, although the underlying behavioral assumptions differ from the assumptions here.

⁸A natural exception arises when brands become synonymous with the product category (e.g., Photoshop, Uber, Velcro), in which case brand recall may dominate value associations.

in which both firms decide to associate their brands with value in their positioning messages, such that $x_0^1 = x_0^2 = 1$. In the associative memory framework, these actions form message $m_0^1 = \left[\begin{array}{ccc} 1 & 0 & \phi \cdot x_0^1 \end{array} \right]' = \left[\begin{array}{ccc} 1 & 0 & \phi \end{array} \right]'$ for firm 1 and $m_0^2 = \left[\begin{array}{ccc} 0 & 1 & \phi \cdot x_0^1 \end{array} \right]' = \left[\begin{array}{ccc} 0 & 1 & \phi \end{array} \right]'$ for firm 2, where the first two elements of each message denote the brand identity of the advertiser (i.e., the brand featured in the advertisement) and the last element depends on the firm's communication decision: it is equal to the recall strength ϕ if value is emphasized, and equal to zero if it is not. The positioning statements initialize the consumer's internal memory state:

$$W_0 = \left[\begin{array}{c} 1 \\ 0 \\ \phi \end{array} \right] \cdot \left[\begin{array}{ccc} 1 & 0 & \phi \end{array} \right] + \left[\begin{array}{c} 0 \\ 1 \\ \phi \end{array} \right] \cdot \left[\begin{array}{ccc} 0 & 1 & \phi \end{array} \right] = \left[\begin{array}{cc|c} 1 & 0 & \phi \\ 0 & 1 & \phi \\ \hline \phi & \phi & 2\phi^2 \end{array} \right] \quad (9)$$

where the lines in the matrix above visually separate the recall strengths of the brands from the recall strength of value (which firms can decide to emphasize) in the bottom right corner. Continuing the example, consider the case where in the second stage, the consumer is exposed to firm 1's advertisement, which again emphasizes value, that is, $x_1^1 = 1$. The advertisement will trigger consumer recall in the form of vector r_1 :

$$r_1 = W_0 \cdot m_1^1 = \left[\begin{array}{cc|c} 1 & 0 & \phi \\ 0 & 1 & \phi \\ \hline \phi & \phi & 2\phi^2 \end{array} \right] \cdot \left[\begin{array}{c} 1 \\ 0 \\ \phi \end{array} \right] = \left[\begin{array}{c} 1 + \phi^2 \\ \phi^2 \\ \phi(1 + 2\phi^2) \end{array} \right] \quad (10)$$

As discussed in the previous section, vector r_1 contains the recall strengths of each component. A concept whose recall strength is greater or equal to threshold θ is remembered by the consumer. In the example above, when threshold θ is very high ($\theta > \phi(1 + 2\phi^2)$) – for example because advertising is not sufficiently effective, or alternatively, it is very costly – firm 1's advertisement is ineffective and fails to prime any memories in the consumer's mind. As a result, the consumer's consideration set remains empty and no trade takes place. When θ is in interval $(1 + \phi^2, \phi(1 + 2\phi^2)]$, the concept of value comes to the consumer's mind upon advertising exposure, but she does not associate it with any brand, similarly leading to no trade. In this parameter space, it is easier for the consumer to recall the communicated benefit than the brand responsible for the advertisement. When θ is intermediate ($\theta \in (\phi^2, 1 + \phi^2]$), upon being exposed to advertisement of firm 1, the consumer is able to recall its brand in association with value, but not the competitor's brand. Because brand 1 is remembered in association with value, the consumer considers it as a relevant alternative for purchase. Finally, when θ is low ($\theta < \phi^2$), firm 1's advertisement also brings to mind

firm 2’s brand. The co-activation follows from the fact that firms associated their brands with value in the positioning stage, thus creating an indirect link between their brands. As a result, in this parameter region firms need to compete more fiercely for the consumer’s business.

We consider pure strategy symmetric subgame-perfect equilibria. If there exist multiple equilibria, we focus on Pareto-efficient outcomes (see, for example, Hossain and Morgan, 2009; Xu and Dukes, 2019).⁹ For expositional purposes, consider first the case in which θ known. We focus the analysis on the parameter space $\theta < \bar{\theta} \equiv 1 + \phi^2$ because, beyond this threshold, firms are unable to induce trade even when they emphasize value in the positioning and advertising stages.

3.1 Deterministic Advertising Effectiveness

Consider the possible communication policy profiles and available deviations starting from the last information set. The equilibrium, starting with the positioning-stage profile, is characterized in the following results.¹⁰

Proposition 1. *When θ is deterministic, firms position their brands in association with value.*

This result follows from the priming nature of associative memory. For advertising to be effective, relevant associations must first be encoded in the consumer’s memory. By linking their brands to value in the positioning stage, firms ensure that subsequent advertising can successfully activate both brand and value at the time of recall. Absent this initial association, later messages are ineffective at generating relevant consideration. Hence, in the positioning stage, $m_1^1 = \begin{bmatrix} 1 & 0 & \phi \end{bmatrix}$ and $m_1^2 = \begin{bmatrix} 0 & 1 & \phi \end{bmatrix}$ are selected by the firms.

We now characterize equilibrium advertising strategies and the resulting market outcomes as a function of parameter θ .

⁹There exists an extensive literature that finds that coordination toward Pareto-efficient outcomes can often be achieved by introducing pre-play communication (see, for example, Blume and Ortmann, 2007). In advertising markets, firms’ strategies often unfold in repeated and public environments, which facilitates tacit coordination on focal outcomes. In this setting, Pareto-efficient equilibria are those that avoid mutually detrimental cross-recall dynamics that intensify competition without increasing the likelihood of consideration. Overall, the outcomes we focus on are both payoff-dominant and behaviorally plausible, providing a natural refinement for equilibrium selection.

¹⁰All proofs are provided in the appendix.

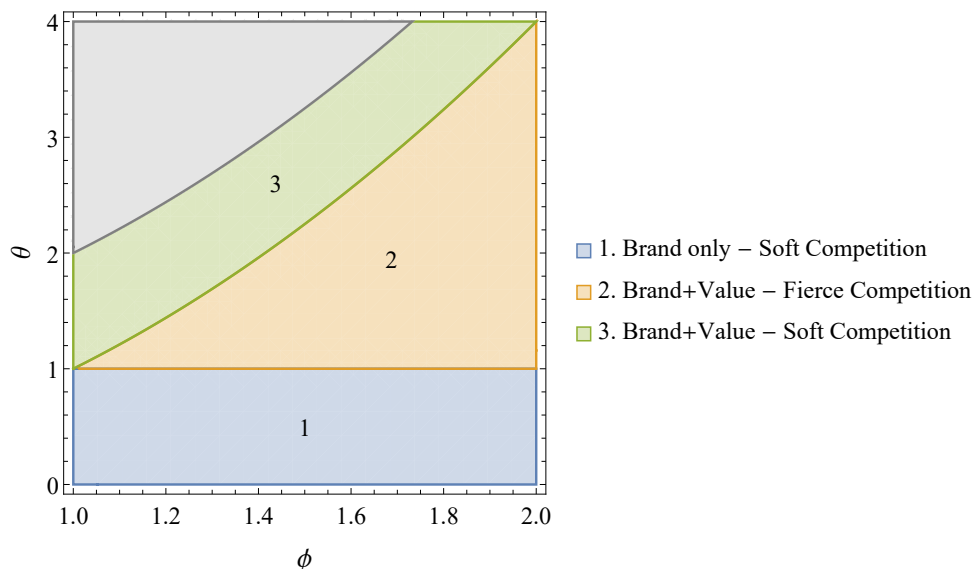
Proposition 2. *When θ is deterministic, firms advertise their brands in isolation if and only if $\theta \leq 1$, and associate their brands with value in the advertising stage if and only if $\theta > 1$.*

The communication policy used by firms during the advertising stage depends on the value of θ . When advertising effectiveness is likely to be low (i.e., θ is high), firms are better off associating their brands with value. In this case, the stimulus from exposure to firm 1's advertisement is equal to $m_1^1 = \begin{bmatrix} 1 & 0 & \phi \end{bmatrix}$, and the stimulus from exposure to firm 2's advertisement is equal to $m_1^2 = \begin{bmatrix} 0 & 1 & \phi \end{bmatrix}$.

On the other hand, when θ is low – in which case advertising effectiveness is interpreted to be high – firms are better off simply referring their brand in the second stage without providing additional information. The stimulus from exposure to firm 1's advertisement is equal to $m_1^1 = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$, and the stimulus from exposure to firm 2's advertisement is equal to $m_1^2 = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}$. These strategies can be interpreted as simple reminders, in which brands are emphasized without associations to specific benefits.

To understand the underlying intuition, we present the parameter regions corresponding to the resulting advertising strategies in Figure 2.

Figure 2: Parameter regions of advertising strategies when θ is known



Note: The colored regions correspond to different equilibrium advertising strategies, and follow the unique equilibrium positioning strategy profile, in which firms associate their brands with value.

In region 1 above ($\theta \leq 1$), advertising communication is quite effective. As a result, it

suffices that firms advertise their own brands for consumers to deem them relevant. In this region, firms strictly prefer not to provide information regarding value, and their advertisements are effective despite featuring only their brands. By keeping advertising uninformative with respect to value, it is possible to induce soft market competition (i.e., a half chance of making a monopoly sale), since advertising exposure only brings to mind the advertiser’s brand and not the rival’s. When advertising effectiveness decreases (region 2, $\theta \in (1, \phi^2]$), uninformative advertising is no longer sufficient to successfully associate firms’ brands with value. In this case, firms are better off explicitly associating their brands with value in the advertising stage. However, doing so also brings the competitor’s brand to mind. The reason is that value is a shared association between the brands. When it is advertised, it acts as a memory backchannel. As a result, any advertising exposure brings both brands to mind, and competition becomes fierce. Fierce competition is unavoidable when θ is intermediate: Emphasizing value is necessary to enter the consideration set, but it simultaneously brings the rival’s brand to mind. Finally, when advertising is less effective (region 3, $\theta \in (\phi^2, 1 + \phi^2]$), firms continue to emphasize value in the advertising stage, but their own advertisements no longer call to mind the competitor’s brand. As a result, profits are higher in this region.

The following proposition formalizes the presentation of the results on the profit and welfare outcomes.

Proposition 3. *When θ is deterministic, firms’ expected equilibrium profits are non-monotonic in θ . They are equal to $\frac{1}{2}\pi^M$ when $\theta \leq 1$ and $\theta > \phi^2$, and are equal to π^C when $\theta \in (1, \phi^2]$. In contrast, consumer welfare is maximized in region $\theta \in (1, \phi^2]$, and is lowest in the complementary region.*

When θ is intermediate, firms face a trade-off: omitting value renders their brand irrelevant, while emphasizing it intensifies competition. In this region both brands are considered regardless of the specific advertising exposure, leading to the lowest degree of profitability. In contrast, the larger consideration set and more intense competition makes consumers better off.

The market characterization above shows that associative memory has important implications for firms’ strategies and market outcomes. While firms benefit from establishing an association between their brands and value early on, their subsequent advertising strategy depends on both the effectiveness of advertising and the relative salience of value compared to the brand. We briefly discuss the role of parameter ϕ , which captures the recall strength of value relative to that of the brands.

We expect ϕ to be high in markets where value is especially salient to consumers, such as

hyped, fashionable, or high-potential markets, such as cryptocurrencies or artificial intelligence. Because these markets attract widespread attention and discussion, their benefits are likely to be highly accessible in consumers’ minds. In region 2, when ‘market hype’ is high ($\phi > \sqrt{\theta}$), firms cannot promote their brands without simultaneously activating their rival’s brand through shared value associations. In contrast, in region 3, $\phi < \sqrt{\theta}$, this spillover effect disappears, allowing firms to emphasize value without triggering competitor recall.

3.2 Stochastic Advertising Effectiveness

The assumption used in the previous section that θ is deterministic is useful to describe the forces in play. However, it is less in line with the real world in which advertising effectiveness is uncertain. Communication processes typically exhibit uncertain outlooks surrounding campaign effectiveness: “Will a given message ‘land’ with the consumer?” “Will advertising effectively affect sales?” From here on, we assume θ is stochastic. We consider the case in which θ follows a cumulative distribution function F with full support $(0, \bar{\theta})$, where $\bar{\theta} \equiv 1 + \phi^2$ as before. The stochastic case allows us to characterize more nuanced outcomes than in the previous section. We establish the unique equilibrium profile in the positioning stage:

Proposition 4. *In equilibrium, firms position their brands in association with value.*

This result generalizes the uniqueness finding in Proposition 1: When θ is uncertain, firms continue to advertise their brands in association with value during the positioning stage. They have no incentive to do otherwise, since failing to associate their brands with value fails to establish brand relevance during the memory priming stage. Even if a single firm decided to advertise value only in the second stage, at that point its rival would have successfully associated its brand with value, gaining a head start in terms of establishing relevance.

Given the uncertainty over parameter θ , the market outcome characterization is more nuanced than in the previous section:

Proposition 5. *In equilibrium, firms advertise their brands in isolation if and only if*

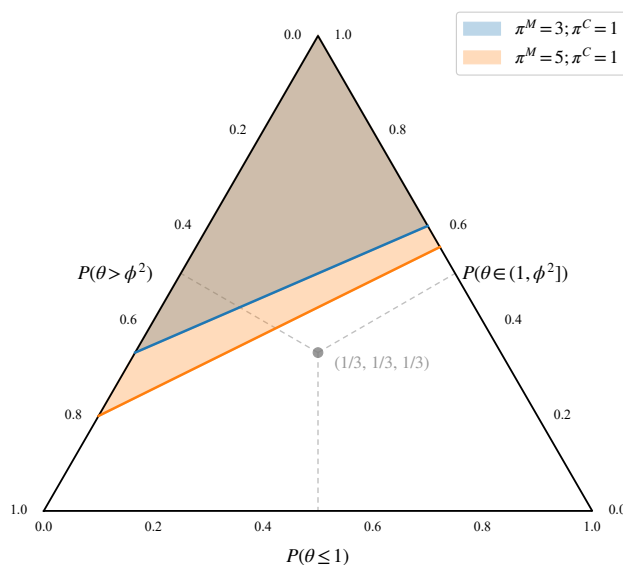
$$\Delta \equiv \frac{P(\theta \leq 1) - P(\theta > \phi^2)}{P(\theta \leq \phi^2)} \pi^M - \pi^C \geq 0 \tag{11}$$

and advertise their brands in association with value otherwise.

Condition (11) summarizes the tensions faced by competing firms. First, note that the

condition is satisfied when π^M is much larger than π^C . This applies to cases of low differentiation/fierce competition, in which having the rival enter the consideration set affects profits severely. In these cases, firms prefer to advertise their brands in isolation to avoid costly cross-recall. Second, the relation between profits in expression (11) depends on the distribution of θ . Comparative statics of the probability distribution are complicated by the fact that probabilities need to sum to one. To obtain further intuition, we depict the region characterized by expression (11) in the ternary plot in Figure 3. The figure illustrates the region in which firms prefer to engage in exclusive advertising (i.e., not emphasize value) as a function of the probability distribution of θ .

Figure 3: Advertising regions as a function of the distribution of θ



Note: The plot above depicts the regions in which firms are better off advertising their brands without emphasizing value. In the complementary regions firms prefer to advertise their brands in association with value. Probabilities are given by the orthogonal distance to the corresponding edge (barycentric coordinates). The numbers on each edge represent the relative split between the probabilities located on opposite edges.

Each edge of the triangle above corresponds to a probability mass equaling zero, and on each vertex, one probability is equal to one.¹¹ The results in Proposition 2 can be observed

¹¹A ternary plot represents three probabilities (p_1, p_2, p_3) that sum to one, so each point in the triangle corresponds to a unique triplet. The three vertices represent the extreme cases in which one probability equals one and the others are zero, while each edge corresponds to one probability being zero. Using barycentric coordinates, each probability is read as the (perpendicular) distance from the opposite edge: moving closer to a vertex increases the corresponding probability. For example, the center of the triangle corresponds to $(1/3, 1/3, 1/3)$, where all probabilities are equal. Thus, any location in the plot directly reflects the relative magnitudes of the three probabilities.

in Figure 3: When $P(\theta \leq 1)$ is equal to one (top vertex of Figure 3), firms prefer to advertise their brands in isolation, and when $P(\theta \leq 1)$ is equal to zero (bottom edge of the plot above), firms prefer to advertise their brands in association with value.

At intermediate values of $P(\theta \leq 1)$, the equilibrium strategies depend on both the remaining probability masses. Consider an imaginary horizontal line crossing point $(1/3, 1/3, 1/3)$. As the line moves to the right, $P(\theta \leq 1)$ remains constant, and probability mass shifts from $P(\theta > \phi^2)$ to $P(\theta \in (1, \phi^2])$, that is, θ is more likely to be high and less likely to be intermediate. The positive slope in the region boundaries depicted in Figure 3 mean that at constant intermediate levels of $P(\theta \leq 1)$, as $P(\theta > \phi^2)$ increases, the equilibrium transitions from firms advertising only their brands to also advertising value. This allows them to stay relevant, despite the low advertising effectiveness. On the other hand, as $P(\theta \in (1, \phi^2])$ increases (and $P(\theta > \phi^2)$ reduces), mentioning value is more likely to bring the rival's brand to mind, and the equilibrium transitions to employing brand reminders.

Overall, firms are better off advertising only their brands when advertising is very likely to be effective ($P(\theta \leq 1)$ high), and prefer to associate with value when advertising is very unlikely to be effective ($P(\theta \leq 1)$ low). When $P(\theta \leq 1)$ is intermediate, then firms advertise in association with value if and only if there is a high enough change that advertising effectiveness is very low ($P(\theta > \phi^2)$ high and $P(\theta \in (1, \phi^2])$ low enough).

The following result summarizes firms' equilibrium profitability as a function of the model parameters:

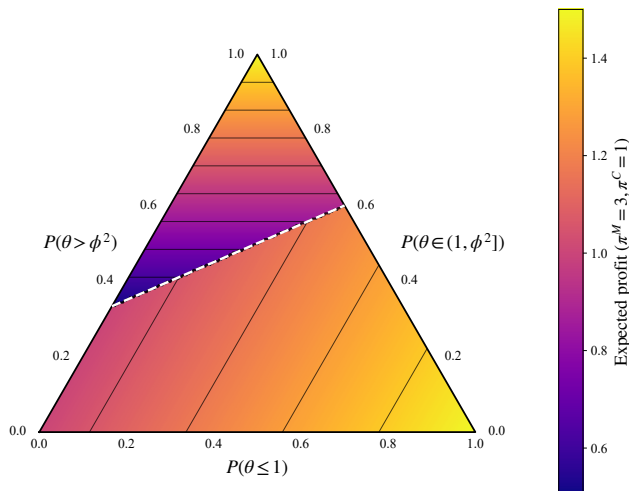
Proposition 6. *In equilibrium, firms' ex-ante equilibrium profits are given by:*¹²

$$\pi = \begin{cases} \frac{1}{2}P(\theta \leq 1) \pi^M, & \Delta > 0 \\ P(\theta \leq \phi^2) \pi^C + \frac{1}{2}P(\theta > \phi^2) \pi^M, & \Delta \leq 0 \end{cases}$$

Figure 4 presents the equilibrium profits for some parameter values.

¹²Note that when $\Delta = 0$, two equilibria co-exist, and the Pareto-efficiency refinement selects the one in which firms emphasize value.

Figure 4: Equilibrium profits as a function of the distribution of θ



Note: The plot above depicts firm ex-ante profits as a function of the probability masses of the distribution of θ , with $\pi^M = 3, \pi^C = 1$.

In the equilibrium region in which firms prefer to advertise their brands in isolation ($\Delta \geq 0$), expected profits increase in $P(\theta \leq 1)$, as depicted in Figure 4, because emphasizing only brands is profitable if advertising is very effective or the payoff of exclusive consideration is very high. The horizontal level curves in the top region of Figure 4 imply that in this region, the specific way in which $P(\theta \leq 1)$ increases (whether it is at the expense of $P(\theta \in (1, \phi^2])$ or $P(\theta > \phi^2)$) is irrelevant for firm profitability. This is due to the fact that in this region both firms advertise only their brands, and co-activation does not occur on equilibrium path.

In the lower equilibrium region ($\Delta \leq 0$), the only risk faced by firms is inducing cross-recall. As a result, expected profits increase unambiguously in $P(\theta > \phi^2)$ and decrease in $P(\theta \leq \phi^2)$. The fact that the level curves in Figure 4 are parallel to the left edge of the plot implies that as $P(\theta > \phi^2)$ increases, firms benefit regardless of the specific manner in which the complementary probability mass $P(\theta \leq 1) + P(\theta \in (1, \phi^2])$ decreases.

Overall, the analysis highlights a central tension in advertising under associative memory: communication that enhances brand relevance may also activate competing brands through shared associations. Consequently, optimal strategies depend on the distribution of advertising effectiveness and the likelihood of joint consideration. We next examine how the introduction of utility-irrelevant associations can mitigate cross-recall and alter equilibrium outcomes.

4 Communicating with Irrelevant Associations

Consider now expanding the communication game from the previous section by introducing additional concepts. In addition to value, let us introduce two utility-irrelevant concepts that firms can use in their communication. The reason to focus on the case where there are more utility-irrelevant concepts than relevant ones is because the prior group is arbitrarily large in the real world (e.g., colors, emotions, mascots, etc), whereas utility-relevant concepts are limited to the benefits obtainable through consumption of specific products. The action space of firms is then expanded to a three-element vector $x \in \{0, 1\}^3$. For convenience, we refer to the irrelevant concepts as colors, such that the communication attributes are ordered as ‘value’, ‘red’, and ‘blue’. For example, when firm 1 would like to associate its brand with value and the color red, its action is coded as $x_1^1 = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix}$. Let us continue to define a brand as attaining relevance when both its own recall strength as well as the recall strength of value are sufficiently strong compared to the memory threshold, whereas the recall strengths of the color concepts are irrelevant for the consumer’s judgment of relevance.

The timing of the game is as presented in the previous section. Consider the illustrative case in which firms associate their brands with value as well as with distinct colors during the positioning phase, via the initial communication policies $x_0^1 = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix}'$ and $x_0^2 = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix}'$. These actions induce the memory state:

$$W_0 = \begin{bmatrix} 1 \\ 0 \\ \phi \\ \phi \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & \phi & \phi & 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ \phi \\ 0 \\ \phi \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & \phi & 0 & \phi \end{bmatrix} = \left[\begin{array}{cc|ccc} 1 & 0 & \phi & \phi & 0 \\ 0 & 1 & \phi & 0 & \phi \\ \hline \phi & \phi & 2\phi^2 & \phi^2 & \phi^2 \\ \phi & 0 & \phi^2 & \phi^2 & 0 \\ 0 & \phi & \phi^2 & 0 & \phi^2 \end{array} \right] \quad (12)$$

Above, the positioning messages received by the consumer are prepended by the firm-brand identifiers and, as before, the recall strengths of the advertised concepts are equal to ϕ . For clarity, the memory matrix presented above is divided by lines separating the recall strengths of brands identities from those of the remaining concepts. We continue to focus on subgame-perfect equilibria, and allow firms to associate their brands with any combination of the three attributes in each stage. One can then obtain the following result:

Proposition 7. *In equilibrium, firms are better off positioning their brands in association*

with value and a distinct color if and only if

$$\Delta = \frac{P(\theta \leq 1) - P(\theta > \phi^2)}{P(\theta \leq \phi^2)} \pi^M - \pi^C \geq 0 \quad (13)$$

In this case, firms emphasize only their own color during the advertising stage. When $\Delta < 0$, firms are better off positioning their brands in association with value and both colors, and communicate a single concept in the advertising stage.¹³

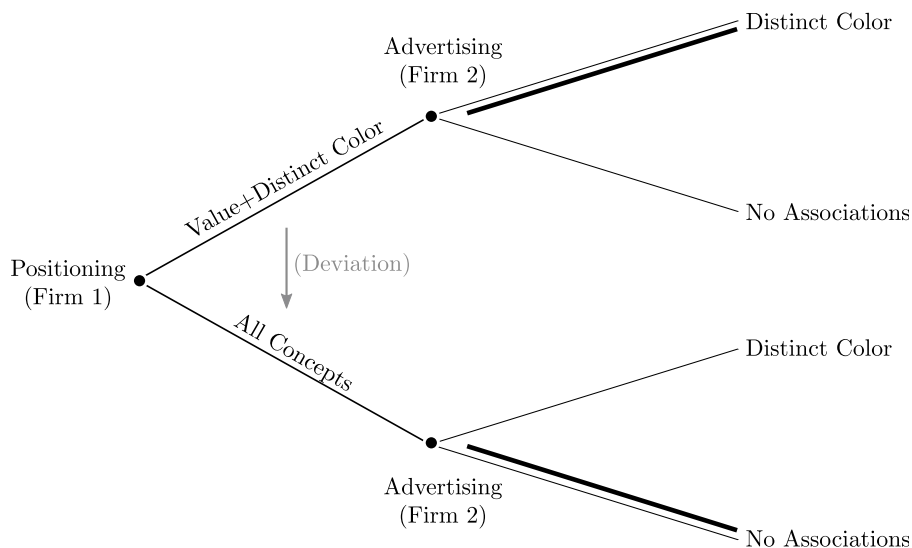
The result above stands out for a few reasons. First, once firms are allowed to include utility-irrelevant concepts in their communication, they will do so despite the apparent irrelevance. Second, unlike the case characterized in Proposition 4, in which firms always want to associate their brands with value, two outcomes result when firms' action space is augmented: Firms may emphasize value and a distinct color, or they may emphasize all concepts during the positioning stage. The latter outcome is relatively intuitive: Because a stronger emphasis always increases recall, there is a region of the parameter space in which firms would like to associate all possible available content with their brand. However, a different outcome may also arise: In region $\Delta \geq 0$, firms associate their brands with value and a distinct color during the positioning phase to avoid inducing cross-recall. This result is the more significant as the consumer we consider does not exhibit bandwidth limitations.

The result that firms may prefer to position their brands with distinct colors is also surprising because it relies on the fact that firms do not benefit from unilaterally deviating from this equilibrium by covering all concepts during the positioning phase. Given that they can always emphasize a subset of concepts during the advertising stage, it is at first unclear why they would not want to cover as much of the conceptual space as possible in the first stage.

Figure 5 illustrates a few actions of the game, in its extensive form.

¹³Note that this region characterization coincides with the one in Proposition 4, although the concepts under consideration are different.

Figure 5: Extensive form game representing the rival's advertising reaction to positioning deviation

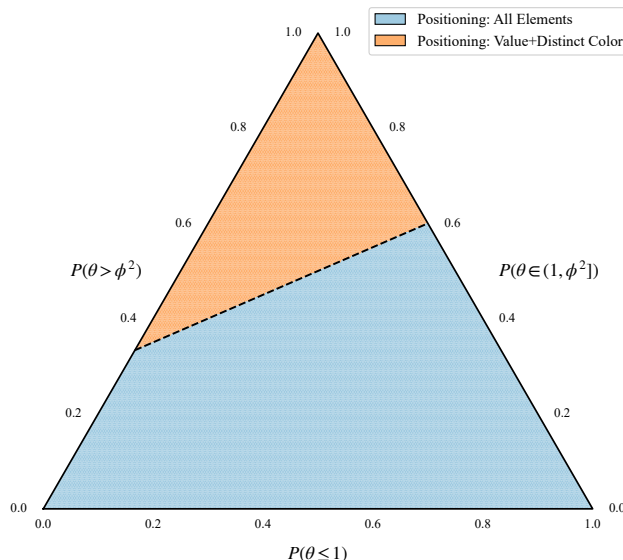


Note: The figure represents only a part of the full extensive form game, namely, the positioning decision of firm 1 and the advertising decision of firm 2. Taking the down action represents a deviation from the equilibrium in which both firms position their brands in association with value and a distinct color. The highlighted branches represent firm 2's advertising best-response when $\Delta \geq 0$.

When firm 1 extends its brand association from value and a distinct color to also include its competitor color, it moves from the top to the bottom branch. In doing so, it induces a reaction by firm 2: The fact that firm 1 has covered all concepts means that firm 2's advertisement may be more likely to induce cross recall. When $\Delta \geq 0$, firm 2 is better off emphasizing its brand with no associations, in order to mitigate the possibility of cross recall. This advertising reaction supports the equilibrium in which firms opt to associate their brands with value and a distinct color in the positioning phase, because it is no longer profitable for firm 1 to cover all concepts during the positioning phase.

The equilibrium positioning regions are depicted in Figure 6.

Figure 6: Positioning regions as a function of the distribution of θ



Note: The plot above depicts firm the positioning equilibrium regions as a function of the probability masses of the distribution of θ , with $\pi^M = 3, \pi^C = 1$.

When $P(\theta \leq 1)$ is high (top vertex of the triangle above), firms prefer to position their brands in association with value and a distinct color. In this region, firms do not want to deviate to mention all concepts during the positioning stage to try to induce cross-recall when the customer is exposed to the rival's advertisement. The reason is that this would induce the rival to advertise only its brand, thwarting the effort while still securing a high chance that its brand is considered. In contrast, when $P(\theta \leq 1)$ is low (lower region of the triangle), firms are better off emphasizing all concepts in the positioning stage. In this case, focusing on distinct colors is no longer sustainable, since during the advertising stage firms will prefer to emphasize more concepts in order to induce direct recall.

Finally, when $P(\theta \leq 1)$ is intermediate (and constant), the positioning equilibrium switches from focusing on value and distinct colors to spanning all concepts, as $P(\theta > \phi^2)$ increases and $P(\theta \in (1, \phi^2])$ decreases. In this case, when $P(\theta > \phi^2)$ is low and $P(\theta \in (1, \phi^2])$ is high, the probability of being able to induce exclusive recall is very low, and so firms are better off focusing on value and distinct colors in their positioning. As $P(\theta > \phi^2)$ increases and $P(\theta \in (1, \phi^2])$ decreases, the likelihood of being able to induce cross-recall also increases, and for that reason firms emphasize all concepts during the positioning phase.

Advertising strategies also change across equilibria. When firms prefer to focus on value and distinct colors in the first stage, they follow up by advertising only their distinct colors in the second stage. This maximizes the strength of their own advertisements, effectively

splitting the market with the competitor (i.e., each advertisement only brings to mind the advertiser’s brand). When firms emphasize all elements in the first stage, they follow up by advertising only one element in the second. Doing so ensures their own advertisements induce recall of their brands, while minimizing the probability of their own advertisement inducing recall of their rival’s brand.

It remains to characterize firms’ profits and consumer welfare:

Proposition 8. *In equilibrium, firms’ ex-ante equilibrium profits are given by:*

$$\pi = \begin{cases} \frac{\pi^M}{2}, & \Delta \geq 0 \\ P(\theta \leq \phi^2) \pi^C + \frac{1}{2} P(\theta > \phi^2) \pi^M, & \Delta < 0 \end{cases} \quad (14)$$

Consumer welfare is highest in the region in which firms emphasize all concepts ($\Delta < 0$), in which cross-recall may occur in equilibrium.

In the first case ($\Delta \geq 0$) outlined above, firms effectively split the market, each firm’s advertisement bringing to mind only the advertiser’s brand. In the second case ($\Delta < 0$), cross-recall is part of equilibrium: When a firm’s advertisement is shown to the customer, both brands come to mind with probability $P(\theta \leq \phi^2)$.

Note that firms are better off in the first region ($\Delta \geq 0$), in which they are able to split the market and induce exclusive consideration. In this region, they also benefit from being able to employ utility-irrelevant concepts in their communication. Comparing with Proposition 6 reveals that firms become strictly better off when $\Delta \geq 0$, whereas the availability of utility-irrelevant concepts adds no value (but also does not destroy it) in region $\Delta < 0$.

The fact that firms employ utility-irrelevant concepts in equilibrium (region $\Delta \geq 0$) is aligned with real-world phenomena. For example, Tiffany & Co. has long been associated with quality and prestige, as well as with its distinctive robin’s-egg blue. The firm continues to rely on this color and has trademarked it. It may be that the specific color chosen by Tiffany & Co. is especially appealing to its target audience, but it is perhaps more likely that the repeated use of this color reinforces the brand in consumers’ minds, along with the associated perceptions of quality and prestige. Given its current brand recognition, it suffices that Tiffany & Co. emphasizes its brand and color without explicitly mentioning quality and prestige, to bring the latter concepts to mind. Moreover, this strategy has the added benefit of minimizing the likelihood of its advertising prompting the brands of rivals.

Examples of utility-irrelevant symbols (e.g., mascots, competences, emotions) are commonplace across markets. In the tire industry, for example, firms rely on distinctive visual

identities that become closely tied to their brands: Michelin to the Michelin Man, Continental to a horse, Pirelli to its elongated yellow-and-red lettering, and Bridgestone to its stylized red “B.” These elements do not affect the physical performance of a tire, nor are they likely to directly influence the driving experience. However, through repeated exposure, they become associated with the brand and with value-relevant attributes such as safety, durability, or performance. As a result, firms can rely on these cues to bring such attributes to mind without making explicit quality claims, limiting the likelihood of activating competing brands.

The second equilibrium region ($\Delta > 0$) predicts that firms rely on a set of similar cues rather than maximally differentiating their communication. This pattern is also evident in real-world markets. In the fast-food industry, logos frequently employ high-contrast color schemes: McDonald’s and In-N-Out Burger both rely on red and yellow palettes, while Five Guys uses similarly bold red lettering. In the technology sector, IBM, Dell, and Intel converge on blue, sans-serif wordmarks. Likewise, in insurance, Aflac and GEICO both employ animal mascots. Many other examples show that firm communication choices do not always attempt to achieve maximal differentiation. Instead, firms navigate a shared conceptual space, balancing the benefits of recognizability against the risk of cross-recall.

5 Conclusion

This paper develops a tractable framework to study advertising and competition when consumers rely on associative memory. We draw on insights from the cognitive psychology literature, which shows that communication can shape what is recalled in consumers’ minds. In the model, advertising reinforces links between brands and concepts through Hebbian learning, thereby influencing which brands and attributes are retrieved when a purchase opportunity arises.

The analysis helps explain several puzzles in advertising markets. First, firms may be better off engaging in uninformative advertising. Once brands are established as valuable in consumers’ minds, firms may optimally rely on messages that merely remind consumers of the brand, without explicitly emphasizing value (e.g., simple calls to action and reminder advertising). Such communication can sustain relevance while limiting the number of alternatives that come to mind, thereby softening competition.

When advertising effectiveness is intermediate, firms may instead benefit from emphasizing value associations. However, the results indicate that this strategy can also open a backchannel in memory: mentioning a desirable benefit – such as convenience or speed – may bring to mind competing brands that share similar benefits. In this region, the con-

sumer contemplates a larger consideration set, which intensifies competition and improves her outcomes through greater choice and lower prices. Finally, when advertising is relatively ineffective, emphasizing value becomes beneficial – and may even be necessary – to induce trade.

We find that firms can mitigate the competitive pressures arising in the intermediate region of advertisement effectiveness by introducing utility-irrelevant cues in their communication. Although these cues do not affect consumption utility, they can reduce the likelihood of cross-brand recall. To accomplish this, firms first associate their brand with both value and a distinctive, irrelevant cue. In subsequent communication, the cue alone can reactivate the brand’s relevance without triggering recall of competing brands.

This logic helps explain why much advertising appears uninformative in the conventional sense. Brand names, logos, colors, mascots, or stylistic elements may perform important competitive functions even though they do not directly affect utility. These cues become valuable not because they convey new information, but because they reactivate the brand together with value-relevant associations, while being less likely than explicit claims to evoke competing brands. Hence, our analysis provides a rationale for firms’ investments in symbolic and aesthetic brand elements that may initially appear peripheral to the product. The model also predicts that, as markets mature, firms shift from emphasizing value toward greater reliance on such cues, which soften competition and may reduce consumer welfare.

The analysis has considered the case in which associative memory can always be strengthened through communication. In reality, firms compete for consumers’ limited attention and memory, which introduces additional strategic considerations. When cognitive resources are constrained, additional communication may crowd out existing associations or dilute their strength. This may strengthen incentives to rely on distinct cues that avoid cross-recall, but may also encourage broader communication strategies that attempt to secure a place in limited memory. These opposing forces suggest that optimal communication balances the breadth of associations with their distinctiveness, and that both differentiation and alignment across cues may arise in equilibrium, consistent with the patterns highlighted in our analysis.

The paper also contributes to the persuasion literature. Much of this literature focuses on how communication affects beliefs – through disclosure, signaling, or inference. In contrast, persuasion in this framework operates through recall and consideration. Consumers need not infer hidden quality or reason about firms’ incentives. Instead, communication is effective insofar as it makes a brand and a relevant concept jointly accessible in memory. This perspective is particularly useful for understanding settings in which advertising appears effective despite conveying little new information.

Several avenues for future research remain open. One important extension concerns advertising targeting, coverage, and reach. The analysis considers consumer behavior upon exposure to communication, but firms differ in their ability to achieve broad exposure. Another promising direction involves extending the model to multidimensional value, allowing firms to compete across multiple relevant attributes. Finally, empirical analysis may help understand how associative memory and also attention constraints shape equilibrium communication strategies.

6 Appendix

Proposition 1

We consider all symmetric positioning profiles and subsequent advertising strategies.

- **Positioning brand in association with value**

First, consider the candidate first-stage memory $W_0 = \left[\begin{array}{cc|c} 1 & 0 & \phi \\ 0 & 1 & \phi \\ \hline \phi & \phi & 2\phi^2 \end{array} \right]$, induced by the positioning strategy $x_0^1 = 1$ and $x_0^2 = 1$. Because the consumer only receives one advertisement, the advertising decision of one firm is independent of the decision of the other in the second stage. Hence, for the second stage, it suffices to analyze the case in which firms act as if the consumer receives only their own advertisement. Given W_0 , the recall strengths induced by firm 1's advertisement in the second period are given by:

$$r_1 = \begin{bmatrix} 1 & 0 & \phi \\ 0 & 1 & \phi \\ \phi & \phi & 2\phi^2 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ x_1^1\phi \end{bmatrix} = \begin{bmatrix} 1 + x_1^1\phi^2 \\ x_1^1\phi^2 \\ \phi(1 + 2x_1^1\phi^2) \end{bmatrix} \quad (15)$$

where x_1^1 is the firm's binary decision of whether to emphasize value in its second-stage advertisement. Table 1 lists the recall strengths induced by the available advertising messages to Firm 1.

Table 1: Recall strengths as a function of firm 1's advertising message

| Id | Advertising Strategy | Recall strengths: | | |
|----|----------------------|-------------------|----------|---------------------|
| | | Firm 1 | Firm 2 | Value |
| 1 | Brand only | 1 | 0 | ϕ |
| 2 | Brand + Value | $1 + \phi^2$ | ϕ^2 | $\phi(1 + 2\phi^2)$ |

Above, advertising the firm's own brand in association with value (strategy 2 above) reinforces all components more strongly than advertising the brand only. On one hand this is positive for the advertiser, since it may be able to make its brand relevant in the mind of the consumer. On the other hand, depending on the value of θ , advertising value may also lead the consumer to remember the rival's brand. Because advertising the brand alone never induces competitor recall, it is the robust outcome provided it can induce recall of the focal brand, in association with value. Therefore, when $\theta \leq 1$, firms prefer to advertise only their

own brands. This ensures a 50% monopoly likelihood for each and a 0% chance of inducing recall of the competitor.

When $\theta > 1$, advertising the brand alone cannot induce relevant recall, since consumers will recall value but fail to associate it with the advertising brand. In this case, the firm's only hope for attracting consumers is advertising its own brand together with value. The market outcomes for the case in which θ is known are presented in Table 2:

Table 2: Market outcomes

| Region | Parameter Region | Advertising Equilibrium | Profits | Competition Level |
|--------|-----------------------------------|-------------------------|--------------------|-------------------|
| 1 | $\theta \leq 1$ | Brand Only | $\frac{1}{2}\pi^M$ | Soft |
| 2 | $\theta \in (1, \phi^2]$ | Brand + Value | π^C | Fierce |
| 3 | $\theta \in (\phi^2, 1 + \phi^2]$ | Brand + Value | $\frac{1}{2}\pi^M$ | Soft |
| 4 | $\theta > 1 + \phi^2$ | — | No trade | |

Consider potential unilateral deviations in the first stage, which consist of a firm not emphasizing value. Let $m_0^1 = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}'$ and $m_0^2 = \begin{bmatrix} 0 & 1 & \phi \end{bmatrix}'$. The induced initial memory state is equal to $W_0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & \phi \\ 0 & \phi & \phi^2 \end{bmatrix}$, and the recall strengths induced by firm 1's advertisement in the second period, as parameterized by binary parameters x_1^1 , is given by:

$$r_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & \phi \\ 0 & \phi & \phi^2 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ x_1^1 \phi \end{bmatrix} = \begin{bmatrix} 1 \\ x_1^1 \phi^2 \\ x_1^1 \phi^3 \end{bmatrix} \quad (16)$$

Table 3 lists the recall strengths induced by the available advertising messages to Firm 1, following it not emphasizing quality in its positioning.

Table 3: Recall strengths as a function of firm 1's advertising message

| Recall strengths: | | | | |
|--------------------------|----------------------|--------|----------|----------|
| Id | Advertising Strategy | Firm 1 | Firm 2 | Value |
| 1 | Brand only | 1 | 0 | 0 |
| 2 | Brand + Value | 1 | ϕ^2 | ϕ^3 |

If firm 1 features only its brand in the advertisement stage, it cannot induce sales, since the recall strength of value associated with strategy 1 above is equal to zero. On the other hand, as long as $\theta \leq 1$, the firm induces a duopoly upon its advertisement exposure, since

$\phi^2 > 1$. In both cases, the firm is strictly better off emphasizing value in the positioning stage. Hence, positioning brands in association with value is a subgame perfect equilibrium.

• **Positioning brand with no association with value**

In the remaining symmetric positioning strategy, neither firm emphasizes value. In this

case, the initial memory state $W_0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ is induced by the positioning strategies

$x_0^1 = 0$ and $x_0^2 = 0$. The recall induced by firm 1's advertisement in the second period, as parameterized by binary parameter x_1^1 , is given by:

$$r_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ x_1^1 \phi \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad (17)$$

In this case, firms can never induce value recall in the second period, and so they are always worse off not positioning their brands alongside value. Moreover, it is easy to show that they benefit from unilaterally deviating to emphasize value in their positioning. Hence, the only symmetric equilibrium positioning strategy is to associate the respective brands with value.

Proposition 2

Proposition 2 follows directly from the results obtained in the previous section, as presented in Table 2.

Proposition 3

Proposition 3 follows directly from the results obtained in the previous section, as presented in Table 2.

Proposition 4

Consider the case in which firms position their brands in association with value in the first stage. Referring to Table 1, and considering symmetric strategies, suppose both firms opt for advertising strategy 1 (advertise only their brands). In this case, they earn expected profits of:

$$E\pi^{0,0} = \frac{1}{2}P(\theta \leq 1)\pi^M \quad (18)$$

If a firm deviates to advertise its brand in association with value, it earns

$$E\pi^{v,0} = \frac{1}{2}P(\theta \leq \phi^2)\pi^C + \frac{1}{2}P(\theta > \phi^2)\pi^M \quad (19)$$

Hence, an equilibrium in which both firms advertise only their brands occurs if and only if:

$$\frac{P(\theta \leq 1) - (P(\theta > \phi^2))}{P(\theta \leq \phi^2)}\pi^M - \pi^C \geq 0 \quad (20)$$

Similarly, it is easy to show that firms prefer to advertise their brands in association with value when the inequality above is not satisfied.

The other candidate symmetric positioning equilibrium profile has firms communicating only their brands in the first stage. In this case, it is straightforward to show that firms are better off unilaterally deviating to emphasize value in the positioning stage, in order to induce trade with positive probability.

Proposition 5

Proposition 5 follows directly from the results obtained in the previous section.

Proposition 6

Proposition 6 follows directly from the results obtained in the proof of Proposition 4.

Proposition 7

Proposition 7 establishes equilibrium in a game with an augmented action space, relative to the game analyzed in Proposition 1. Candidate profiles that are not equilibria in the original game cannot be part of the equilibrium set in the augmented game, so one can rule out the candidate equilibrium profile in which firms position their brands without associating them with other concepts. Consider then the following positioning strategies: i) position the brand in association with value and a distinct color; ii) position the brand in association with value only; iii) position the brand in association with value and a common color; iv) position the brand in association with both colors; v) position the brand in association with value and both colors.

- **i) Position brand in association with value and distinct color from competitor**

When firm 1 associates its brand with value and the color red, and firm 2 associates its brand with value and the color blue, the initial memory state is given by:

$$W_0 = \left[\begin{array}{cc|ccc} 1 & 0 & \phi & \phi & 0 \\ 0 & 1 & \phi & 0 & \phi \\ \hline \phi & \phi & 2\phi^2 & \phi^2 & \phi^2 \\ \phi & 0 & \phi^2 & \phi^2 & 0 \\ 0 & \phi & \phi^2 & 0 & \phi^2 \end{array} \right] \quad (21)$$

and the recall strength vector, as a function of firm 1's advertising strategy in the second period (conditional on its advertisement being shown to the customer), is given by:

$$r_1 = \left[\begin{array}{cc|ccc} 1 & 0 & \phi & \phi & 0 \\ 0 & 1 & \phi & 0 & \phi \\ \hline \phi & \phi & 2\phi^2 & \phi^2 & \phi^2 \\ \phi & 0 & \phi^2 & \phi^2 & 0 \\ 0 & \phi & \phi^2 & 0 & \phi^2 \end{array} \right] \cdot \begin{bmatrix} 1 \\ 0 \\ x_{1,v}^1 \phi \\ x_{1,r}^1 \phi \\ x_{1,b}^1 \phi \end{bmatrix} = \begin{bmatrix} 1 + (x_{1,v}^1 + x_{1,r}^1) \phi^2 \\ (x_{1,v}^1 + x_{1,b}^1) \phi^2 \\ \phi + (2x_{1,v}^1 + x_{1,r}^1 + x_{1,b}^1) \phi^3 \\ \phi + (x_{1,v}^1 + x_{1,r}^1) \phi^3 \\ (x_{1,v}^1 + x_{1,b}^1) \phi^3 \end{bmatrix} \quad (22)$$

where $x_{1,v}^1$, $x_{1,r}^1$, and $x_{1,b}^1$ are the binary elements of the advertising action x_1^1 by firm 1, corresponding to value, red and blue. Table 4 presents the recall strengths induced by the available action profiles:

Table 4: Recall strengths as a function of firm 1's advertising message

| Id | Advertising Strategy | Recall strengths: | | |
|-------------------------------------|-----------------------------|-------------------|-----------|---------------------|
| | | Firm 1 | Firm 2 | Value |
| Not including rival's color: | | | | |
| 1 | Brand only | 1 | 0 | ϕ |
| 2 | Brand + Value | $1 + \phi^2$ | ϕ^2 | $\phi(1 + 2\phi^2)$ |
| 3 | Brand + Own Color | $1 + \phi^2$ | 0 | $\phi(1 + \phi^2)$ |
| 4 | Brand + Value + Own Color | $1 + 2\phi^2$ | ϕ^2 | $\phi(1 + 3\phi^2)$ |
| Including rival's color: | | | | |
| 5 | Brand + Rival Color | 1 | ϕ^2 | $\phi(1 + \phi^2)$ |
| 6 | Brand + Value + Rival Color | $1 + \phi^2$ | $2\phi^2$ | $\phi(1 + 3\phi^2)$ |
| 7 | Brand + Value + Both Colors | $1 + 2\phi^2$ | $2\phi^2$ | $\phi(1 + 4\phi^2)$ |
| 8 | Brand + Both Colors | $1 + \phi^2$ | ϕ^2 | $\phi(1 + 2\phi^2)$ |

It is useful to order the recall strengths in the table above, to obtain an ordering of the concepts that are recalled as θ increases from zero. Below, threshold 'flips' are presented in

bold. These occur beyond the element $1 + \phi^2$,

$$\phi > \phi_{(1)} : \phi^2 < 1 + \phi^2 < 2\phi^2 < \mathbf{1} + \mathbf{2}\phi^2 < \phi (\mathbf{1} + \phi^2) < \phi (1 + 3\phi^2) < \phi (1 + 4\phi^2) \quad (23)$$

$$\phi < \phi_{(1)} : \phi^2 < 1 + \phi^2 < 2\phi^2 < \phi (\mathbf{1} + \phi^2) < \mathbf{1} + \mathbf{2}\phi^2 < \phi (1 + 3\phi^2) < \phi (1 + 4\phi^2) \quad (24)$$

where $\phi_{(1)} \simeq 1.755$ is the real root of $x(x^2 - 2x + 1) = 1$.

Note that each firm can ensure its advertisement induces recall of only its brand with certainty, by advertising its brand in association with the color it emphasized during the positioning stage (action 3, above). A deviation that can make a firm strictly better off would need to accomplish two things: The firm's advertisement should induce exclusive recall of its own brand, and the rival's advertisement should induce recall of the focal firm's advertisement with strictly positive probability. Hence, we check the deviation of firm 1 positioning its brand alongside value and both colors. By doing so, firm 1 enhances its brand strength, and may be able to induce recall even when the consumer is exposed to the rival's brand.

In this case, the induced memory state from firm 1 emphasizing all elements and firm 2 emphasizing value and the color blue is given by:

$$W_0 = \left[\begin{array}{cc|ccc} 1 & 0 & \phi & \phi & \phi \\ 0 & 1 & \phi & 0 & \phi \\ \hline \phi & \phi & 2\phi^2 & \phi^2 & 2\phi^2 \\ \phi & 0 & \phi^2 & \phi^2 & \phi^2 \\ \phi & \phi & 2\phi^2 & \phi^2 & 2\phi^2 \end{array} \right] \quad (25)$$

and the recall induced by each advertisement exposure is given by:

$$r_1 = \left[\begin{array}{c} 1 + (x_{1,b}^1 + x_{1,r}^1 + x_{1,v}^1) \phi^2 \\ (x_{1,v}^1 + x_{1,b}^1) \phi^2 \\ \dots \end{array} \right], \quad r_2 = \left[\begin{array}{c} (x_{1,b}^2 + x_{1,r}^2 + x_{1,v}^2) \phi^2 \\ 1 + (x_{1,v}^2 + x_{1,b}^2) \phi^2 \\ \dots \end{array} \right] \quad (26)$$

where $x_{1,v}^1$, $x_{1,r}^1$, and $x_{1,b}^1$ are the binary components of firm 1's advertising strategy and $x_{1,v}^2$, $x_{1,r}^2$, and $x_{1,b}^2$ are the binary components of firm 2's advertising strategy.¹⁴

Inspection of the recall strengths reveals that firm 1 is strictly better off emphasizing the red color in the advertising stage ($x_{1,r}^1 = 1$), because this increases its own brand recall without affecting the recall strength of its rival (who did not emphasize the color red during the positioning stage). By setting $x_{1,v}^1 = x_{1,b}^1 = 0$, firm 1 can ensure its advertisement exposure

¹⁴The last elements of the recall vectors are omitted because they dominate the brand ones.

induces exclusive recall, which is the first best outcome for its advertisement exposure.

As for firm 2, it has no incentive to emphasize $x_{1,r}^2$, because this element only increases the probability that the rival firm is considered. As for the remaining options, if it emphasizes all elements, firm 2 earns payoff π^C upon its own advertisement exposure (both brands will be recalled with certainty). Firm 2 can do better by emphasizing one element (either $x_{1,v}^2$ or $x_{1,b}^2$): In this case its advertisement induces recall of its brand with certainty. Upon exposure to its advertisement emphasizing either $x_{1,v}^2$ or $x_{1,b}^2$, firm 2's expected profit is equal to: $P(\theta \leq \phi^2) \pi^C + P(\theta > \phi^2) \pi^M$. Finally, when firm 2 advertises only its brand, its advertisement exposure earns her $P(\theta \leq 1) \pi^M$.

The profile in which firm 1 achieves cross-recall occurs if and only if firm 2 prefers to emphasize either value or the color blue in the advertising stage, that is, when:

$$P(\theta \leq \phi^2) \pi^C + P(\theta > \phi^2) \pi^M \geq P(\theta \leq 1) \pi^M \quad (27)$$

When the condition above holds, firm 1 is better off deviating in the positioning stage by emphasizing all elements. Firm 1's expected profit is equal to:

$$\pi_{Dev}^1 = \frac{1}{2} \pi^M + P(\theta \leq \phi^2) \pi^C \quad (28)$$

No other deviations can reinforce firm 1's brand to produce this consideration result. This implies that positioning brands along side value and distinct colors (followed by advertising own colors) is a subgame perfect equilibrium if

$$\frac{P(\theta \leq 1) - P(\theta > \phi^2)}{P(\theta \leq \phi^2)} \pi^M \geq \pi^C \quad (29)$$

- **ii) Position the brand in association with value only**

In this case, the induced recall strengths of firm 1's advertisement are given by $r_1 = \left[1 + x_{1,v}^1 \phi^2, x_{1,v}^1 \phi^2, \dots \right]'$, and the recall strengths of the rival's advertisement are symmetric. By deviating to emphasizing value and a distinct color in the positioning stage, the recall strengths of firm 1's advertisement become $r_1' = \left[1 + (x_{1,r}^1 + x_{1,v}^1) \phi^2, x_{1,v}^1 \phi^2, \dots \right]'$. By advertising only the red color in the advertising stage, firm 1 can do strictly better, since its advertisement will induce recall of its own brand but not of the competitor's. Hence, positioning the brand in association with value only is not a subgame perfect equilibrium.

- **iii) Position the brand in association with value and a common color**

In this case, the induced recall strengths of firm 1's advertisement (assume both firms em-

phasize the red color) are given by $r_1 = \left[1 + (x_{1,r}^1 + x_{1,v}^1) \phi^2, (x_{1,r}^1 + x_{1,v}^1) \phi^2, \dots \right]'$, and the recall strengths of the rival's advertisement are symmetric. As in the previous case, by deviating and emphasizing a distinct color, firm 1 can have its advertisement induce recall of only its brand, and do better. Hence, positioning the brand in association with value and a common color is not a subgame perfect equilibrium.

• **iv) Position the brand in association with both colors**

In this case, the recall strength of the value component is always equal to zero, regardless of the advertising strategy. Firms can do better by emphasizing value and enable the probability of trade. Hence, positioning the brand in association with both colors only is not a subgame perfect equilibrium.

• **v) Position the brand in association with value and both colors**

In this case, the induced recall strengths of firm 1's advertisement are given by

$$r_1 = \left[1 + (x_{1,b}^1 + x_{1,r}^1 + x_{1,v}^1) \phi^2, (x_{1,b}^1 + x_{1,r}^1 + x_{1,v}^1) \phi^2, \dots \right]',$$

and the recall strengths of the rival's advertisement are symmetric. In this case, it suffices each firm selects a single component to ensure its brand is recalled upon its own advertisement exposure. Emphasizing more elements does not increase this probability, but it increases the probability the rival is also recalled. Hence, in the tentative advertising equilibrium firms either emphasize zero or one concepts. Considering symmetric outcomes, if firms advertise only their brands in the second stage, their ex-ante profits are equal to

$$\pi^{0,0} = \frac{1}{2} P(\theta \leq 1) \pi^M \quad (30)$$

If a firm deviates and also advertises value (for example), it obtains profit

$$\pi^{v,0} = \frac{1}{2} P(\theta \leq \phi^2) \pi^C + \frac{1}{2} P(\theta > \phi^2) \pi^M \quad (31)$$

Hence, following a positioning in which firms emphasize all elements, the subsequent advertising strategy depends on the relation between the expressions above. Firms are better off advertising only their brands iff

$$\frac{1}{2} P(\theta \leq 1) \pi^M \geq \frac{1}{2} P(\theta \leq \phi^2) \pi^C + \frac{1}{2} P(\theta > \phi^2) \pi^M \quad (32)$$

$$\Leftrightarrow \frac{P(\theta \leq 1) - P(\theta > \phi^2)}{P(\theta \leq \phi^2)} \pi^M \geq \pi^C \quad (33)$$

Can firms do better by deviating unilaterally in the positioning stage? It is easy to show that they cannot. The reason is that by reducing the number of elements emphasized in the positioning stage, the deviating firm is less able to induce recall of its brand upon its own advertisement exposure, with no consequence on the rival's brand recall. For example, if it emphasizes value and the red color, its advertisement recall strengths become $r_1 = \left[1 + (x_{1,r}^1 + x_{1,v}^1) \phi^2, (x_{1,b}^1 + x_{1,r}^1 + x_{1,v}^1) \phi^2, \dots \right]'$. As for the effect on the rival's advertisement, emphasizing fewer concepts in the positioning stage reduces one's own brand recall strength when the rival's advertisement is shown to the consumer. Hence, firms cannot become strictly better off by deviating from positioning their brands in association with value and both colors.

• **Equilibrium Selection**

Table 5 summarizes the game sub-game perfect equilibria.

Table 5: Subgame-Perfect Equilibrium Profiles and Profits

| Profile | Condition | Positioning | Advertising | Ex-ante Profit |
|---------|--|----------------------------|-----------------|--|
| 1 | $\frac{P(\theta \leq 1) - P(\theta > \phi^2)}{P(\theta \leq \phi^2)} \pi^M \geq \pi^C$ | Brand+Value+Distinct Color | Brand+Own Color | $\frac{1}{2} \pi^M$ |
| 2 | $\frac{P(\theta \leq 1) - P(\theta > \phi^2)}{P(\theta \leq \phi^2)} \pi^M \geq \pi^C$ | Brand+All Elements | Brand | $\frac{1}{2} P(\theta \leq 1) \pi^M$ |
| 3 | $\frac{P(\theta \leq 1) - P(\theta > \phi^2)}{P(\theta \leq \phi^2)} \pi^M \leq \pi^C$ | Brand+All Elements | One Element | $P(\theta \leq \phi^2) \pi^C + \frac{1}{2} P(\theta > \phi^2) \pi^M$ |

Consider the set of equilibrium profiles as $\frac{P(\theta \leq 1) - P(\theta > \phi^2)}{P(\theta \leq \phi^2)} \pi^M$ increases. When

$$\frac{P(\theta \leq 1) - P(\theta > \phi^2)}{P(\theta \leq \phi^2)} \pi^M < \pi^C,$$

the unique subgame-perfect equilibrium is profile 3 above. When $\frac{P(\theta \leq 1) - P(\theta > \phi^2)}{P(\theta \leq \phi^2)} \pi^M = \pi^C$, all profiles are part of the equilibrium set, and when $\frac{P(\theta \leq 1) - P(\theta > \phi^2)}{P(\theta \leq \phi^2)} \pi^M > \pi^C$, the equilibrium set is composed of profiles 1 and 2. Note that profiles 1 and 2 apply to the same region, and profile 1 Pareto-dominates profile 2. Hence, profile 2 does not survive refinement. As for the case when $\frac{P(\theta \leq 1) - P(\theta > \phi^2)}{P(\theta \leq \phi^2)} \pi^M = \pi^C$, profile 1 dominates profile 2, which is then always satisfied. Hence, the equilibrium set is given by:

$$Eq. Set = \begin{cases} \text{profile 3,} & \frac{P(\theta \leq 1) - P(\theta > \phi^2)}{P(\theta \leq \phi^2)} \pi^M < \pi^C \\ \text{profile 1,} & \frac{P(\theta \leq 1) - P(\theta > \phi^2)}{P(\theta \leq \phi^2)} \pi^M \geq \pi^C \end{cases}$$

Proposition 8

Proposition 8 follows directly from the results obtained in the proof of Proposition 7.

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